

Compression and collisions of chirped pulses in a dense two-level medium

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Using numerical simulations, we study propagation of linearly-chirped optical pulses in a homogeneously broadened two-level medium. We pay attention to the three main topics – validity of the rotating-wave approximation (RWA), pulse compression, and collisions of counter-propagating pulses. The cases of long and single-cycle pulses are considered and compared with each other. We show that the RWA does not give a correct description of chirped pulse interaction with the medium. The compression of the chirp-free single-cycle pulse is stronger than of the chirped one, while the opposite is true for long pulses. We demonstrate that the influence of chirp on the collisions of the long pulses allows to control the state of the transmitted radiation: the transmission of the chirp-free pulse can be dramatically changed under collision with the chirped counter-propagating one, in sharp contrast to the case when both pulses are chirped. On the other hand, the collisions of the chirped single-cycle pulses can be used for precise control of medium excitation in a narrow spatial region.

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I. INTRODUCTION

Since the discovery of self-induced transparency (SIT) [1, 2], light interaction with two-level quantum media attracted much attention and was discussed in a number of books [3, 4] and review papers [5–8]. This interest is, to a great extent, due to the fundamental importance of the semiclassical two-level model which is one of the basic models of nonlinear optics and laser physics. Besides the SIT itself, there was a deep investigation of other nonlinear effects in two-level media, such as intrinsic (mirrorless) optical bistability [9, 10], influence of local-field correction (near dipole-dipole interactions) on SIT solitons and optical switching [11–13], population control with specially constructed pulses [14], incoherent soliton generation [15], collisions of solitons [16, 17], solitons in periodically modulated two-level media [18–20], etc. A more recent topic is connected with study of few-cycle and sub-cycle pulses in the two-level media when the standard rotating-wave approximation (RWA) turns out to be invalid [21–23]; see also the recent reviews [24, 25] and references therein.

Additional degree of freedom is provided by chirp, i.e. temporal variation of the carrier frequency of the pulse. Influence of chirp on pulse propagation in the two-level medium is under examination, at least, from the 1980s [26]. More recent theoretical studies allowed to find the analytical solutions for a certain class of chirped pulses [27] and investigate the validity of the RWA for the so-called ultrachirped pulses [28], showed the splitting of chirped pulses with particular spectral composition [29], demonstrated the soliton formation from a two-component chirped pulse [30] and the coherent control of spectral shifts [31], analyzed excitation of the medium

with the sub-cycle and single-cycle chirped pulses and formation of sub-cycle solitons [23, 32, 33], etc.

In this paper, we consider some aspects of medium-light interaction in the case of linearly-chirped short pulses. We focus on the three main questions which, as far as we know, were not studied in detail previously – test of the RWA validity, pulse compression and collisions of counter-propagating pulses. We use numerical simulation technique described briefly in Section II to directly verify the validity of the RWA for description of linearly-chirped pulse propagation and to study the compression of such pulses and soliton formation. In Section III, the pulses are suggested to be long enough, so that the RWA violation cannot be connected with the processes on the single-cycle scale studied previously [22]. As to collisions of counter-propagating chirped pulses inside the medium, the present study continues our previous work where interaction of chirp-free pulses in both homogeneously and inhomogeneously broadened two-level media was analyzed [17, 34, 35]. It was shown that, changing intensity of the first pulse, one can effectively control the transmission of the second pulse. Here we study the influence of chirp on collisions of counter-propagating pulses and the possibility to control the parameters of transmitted radiation with the chirped pulses. In Section IV, the case of single-cycle pulses is considered and compared with the results obtained for the long pulses. The paper is completed with the brief Conclusion.

II. MAIN EQUATIONS AND PARAMETERS

We describe light propagation in the homogeneously broadened two-level medium beyond the RWA and the slowly-varying envelope approximation (SVEA) with the Maxwell–Bloch equations as given in our previous publi-

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cation [34]:

$$\frac{\partial^2 \Omega}{\partial \xi^2} - \frac{\partial^2 \Omega}{\partial \tau^2} - 2i \frac{\partial \Omega}{\partial \xi} - 2i \frac{\partial \Omega}{\partial \tau} = 6\epsilon \left(\frac{\partial^2 p}{\partial \tau^2} + 2i \frac{\partial p}{\partial \tau} - p \right), \quad (1)$$

$$\frac{dp}{d\tau} = i\delta p + \frac{i}{2}(\Omega + s\Omega^* e^{-2i(\tau-\xi)})w - \gamma'_2 p, \quad (2)$$

$$\frac{dw}{d\tau} = i(\Omega^* p - \Omega p^*) + is \left(\Omega p e^{2i(\tau-\xi)} - \Omega^* p^* e^{-2i(\tau-\xi)} \right) - \gamma'_1 (w + 1), \quad (3)$$

where $\tau = \omega t$ and $\xi = kz$ are the dimensionless time and distance; $\Omega = (\mu/\hbar\omega)A$ is the dimensionless field amplitude (normalized Rabi frequency); A and p are the complex amplitudes of the electric field and atomic polarization, respectively; w is the inversion (difference between populations of excited and ground states); $\delta = \Delta\omega/\omega = (\omega_0 - \omega)/\omega$ is the normalized frequency detuning; ω_0 is the frequency of atomic resonance; ω is the light carrier frequency; μ is the dipole moment of the quantum transition; $\gamma'_{1,2} = \gamma_{1,2}/\omega$ are the normalized relaxation rates of population and polarization, respectively; $\epsilon = \omega_L/\omega = 4\pi\mu^2 C/3\hbar\omega$ is the dimensionless parameter of interaction between light and matter (normalized Lorentz frequency); C is the concentration (density) of two-level atoms; $k = \omega/c$ is the wavenumber; c is the speed of light, and \hbar is the Planck constant. Asterisk stands for complex conjugation. We introduced here the auxiliary two-valued coefficient s , so that $s = 0$ corresponds to the RWA (absence of “rapidly rotating” terms), while $s = 1$ is used in the general case.

Further, we solve Eqs. (1)–(3) numerically choosing the appropriate value of s . The numerical approach is the same as in our previous publication [34], more details on it can be found in [36]. We perform calculations for the following parameters of the medium and light: the relaxation rates $\gamma_1 = 1$ and $\gamma_2 = 10 \text{ ns}^{-1}$ are large enough, so that we are in the regime of coherent light-matter interaction; the detuning $\delta = 0$ (exact resonance, $\omega = \omega_0$); the central light wavelength $\lambda = 2\pi c/\omega_0 = 0.83 \text{ }\mu\text{m}$; and the strength of light-matter coupling $\omega_L = 10^{11} \text{ s}^{-1} \ll \omega$. For this choice of parameters, the inequality $\Omega\omega \gg \omega_L$ is valid, so that we can neglect here the so-called local field effects [37]. The medium is supposed to be initially in the ground state ($w = -1$).

In this paper, we consider the pulses of Gaussian shape with linear chirp, so that for the incident normalized Rabi frequency (electric field amplitude) we have $\Omega = \Omega_p \exp(-(t - t_0)^2/2t_p^2 + i\beta\omega_0^2 t^2)$, where β is the dimensionless chirp parameter (chirp normalized by ω_0^2), i.e. the instantaneous carrier frequency changes linearly with time as $\omega_i(t) = \omega_0 + 2\beta\omega_0^2 t$. The duration of the pulse t_p is defined through the number of cycles N as $t_p = NT/2\sqrt{\ln 2}$, where $T = \lambda/c$ is the period of electric field oscillations. The parameter t_0 governs the instant

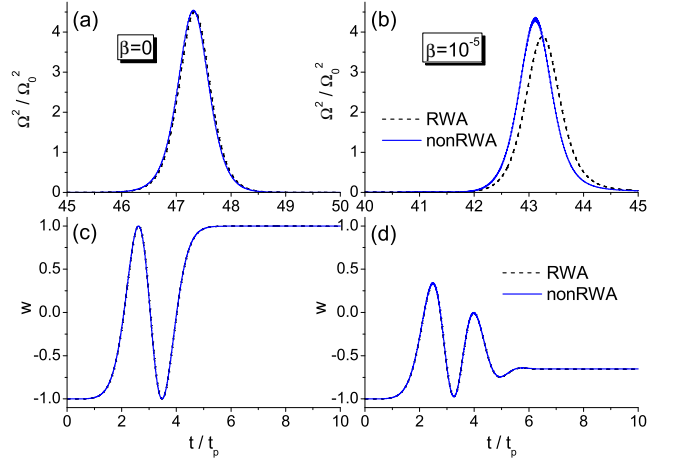


FIG. 1. (Color online) (a, b) The profiles of transmitted radiation and (c, d) inversion dynamics (at the medium entrance) for the incident pulse (a, c) without chirp and (b, d) with the chirp $\beta = 10^{-5}$. The layer thickness is $L = 1000\lambda$, the pulse amplitude is $\Omega_p = 1.5\Omega_0$.

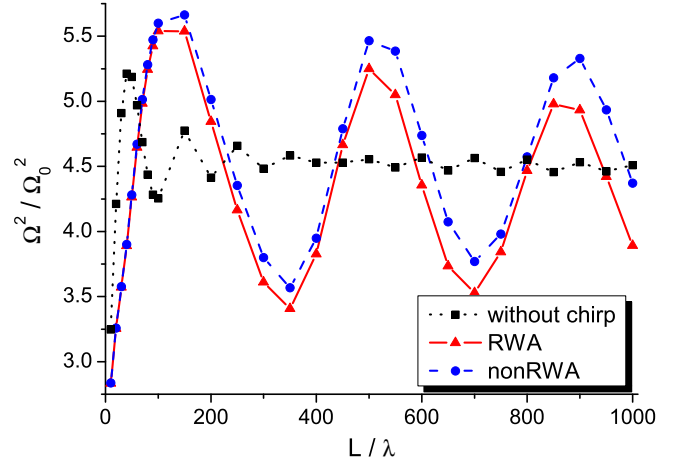


FIG. 2. (Color online) The dependence of the peak intensity of the transmitted pulse calculated for the chirp values of $\beta = 0$ and $\beta = 10^{-5}$. In the latter case, both RWA and non-RWA calculations were performed. The pulse amplitude is $\Omega_p = 1.5\Omega_0$.

of maximum of the pulse intensity (the peak offset). It is important to note that the instantaneous frequency is not equal to ω_0 at the pulse peak: $\omega_i(t)$ grows linearly from ω_0 at $t = 0$ and, at $t = t_0$, differs from this initial frequency more or less significantly. The peak Rabi frequency Ω_p is measured in the units of $\Omega_0 = \lambda/\sqrt{2\pi}ct_p$ corresponding to the chirp-free pulse area 2π .

III. LONG PULSES

In this section, we consider the long pulses with the number of cycles $N = 50$ and the peak offset $t_0 = 3t_p$. We

start with the dynamics of a single chirped pulse in the two-level medium focusing on pulse transmission through layers of different thicknesses. One of our main intentions is to test the applicability of the RWA for description of pulse dynamics. Figure 1 shows the comparison of the intensity profiles and inversions calculated with and without the RWA for the chirp-free pulse and for the pulse with the chirp parameter as large as $\beta = 10^{-5}$. The pulse amplitude $\Omega_p = 1.5\Omega_0$ corresponds to the area 3π (in the chirp-free limit), so that the non-chirped pulse leaves the medium in the fully inverted state [see Fig. 1(c)]. This is not the case for the chirped pulse [Fig. 1(d)]: the medium stays only partially excited, perhaps, because of violation of the resonance conditions due to sweeping of the pulse frequency. It is seen that calculations with the RWA ($s = 0$) and without it ($s = 1$) give essentially the same dynamics of the inversion, but not of the intensity profiles of the transmitted radiation. The RWA works perfectly in the case of chirp-free pulse [Fig. 1(a)], but the results for chirped pulse diverge [Fig. 1(b)]. This difference between the RWA and non-RWA profiles seems to be the result of small deviations from the exact dynamics which accumulate as the pulse propagates in the medium.

This conclusion is corroborated in Fig. 2 where the dependence of the transmitted pulse peak intensity on the medium thickness L is shown. It is clearly seen that the difference between the RWA and non-RWA curves calculated for the pulses with $\beta = 10^{-5}$ grows with L . Calculations using the RWA give lower intensities in comparison with the general model. It is also interesting to compare compression of the pulses with and without linear chirp which can be traced by change of the peak intensity of the pulse. It is seen in Fig. 2 that, for the chirp-free pulse (black squares), the peak intensity rises from $\Omega_p^2 = 2.25\Omega_0^2$ to the maximum of about $5.25\Omega_0^2$ at the comparatively short distance in the medium (less than 100λ) and then, after some oscillations, tends to the quasi-stationary level of about $4.5\Omega_0^2$ (formation of the 2π SIT soliton). These oscillations are much more pronounced for the chirped pulse: though the average level of compression is essentially the same (peak with $4.5\Omega_0^2$), the scatter of data around this mean value allow to obtain at different medium thicknesses strongly differing results (from $3.5\Omega_0^2$ to $5.5\Omega_0^2$). These oscillations relax and, perhaps, result in (quasi)solitonic pulse formation, but much more slowly than in the case of $\beta = 0$. All in all, the chirped pulses may be useful to obtain stronger compressions, especially at larger medium thicknesses than the chirp-free pulses.

Let us consider the situation of colliding counter-propagating pulses and study the influence of chirp on their interaction. We call one of the pulses (forward propagating) the signal pulse, while the counter-propagating one is named the control pulse. We focus on the following question: How the control pulse influences the properties of the signal one? To find the answer, we fix the amplitude of the signal pulse to $\Omega_p^{(s)} = \Omega_0$ (2π pulse) and

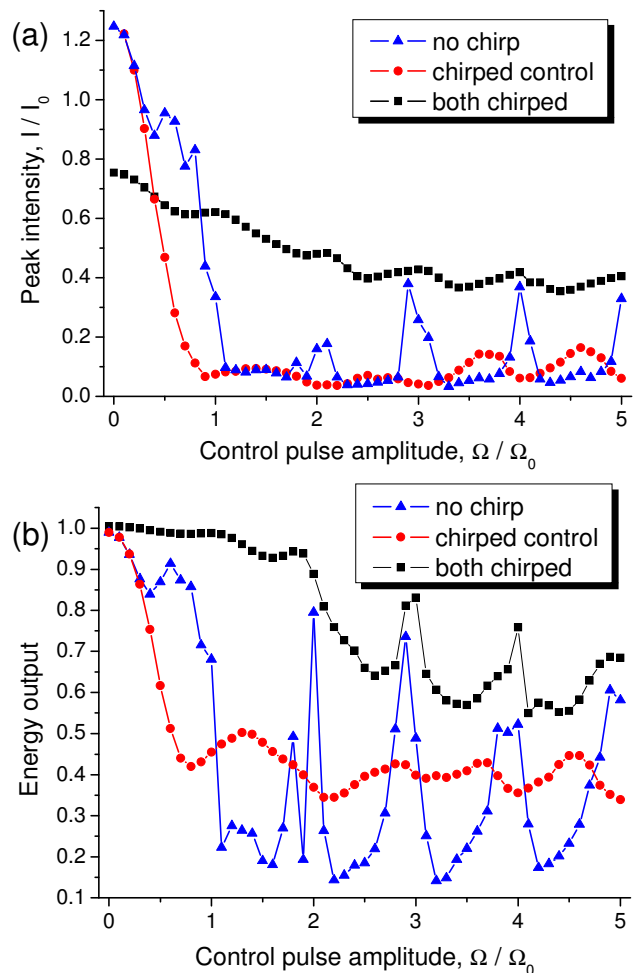


FIG. 3. (Color online) The dependencies of (a) the peak intensity of the signal pulse and (b) the part of its energy at the output on the amplitude of the control (counter-propagating) pulse. Calculations were performed for three cases: (i) both pulses without chirp, (ii) the control pulse is chirped, but the signal one is not, (iii) both pulses are chirped. The chirp is $\beta = 10^{-5}$, the medium thickness $L = 350\lambda$, the signal pulse amplitude $\Omega_p = \Omega_0$.

change the amplitude of the control pulse $\Omega_p^{(c)}$ from 0 to $5\Omega_0$ (area from 0 to 10π). The results of calculations are shown in Fig. 3. We consider three cases: (i) both pulses have no chirp, (ii) the control pulse is chirped, but the signal one is not, (iii) both pulses are chirped.

The starting point is the interaction of chirp-free pulses which was studied in detail in our previous works [17, 22]. The curves for the signal peak intensity and the energy output (the part of signal pulse energy left the medium after the time $100t_p$) as a function of control pulse amplitude in this chirp-free case are shown with blue triangles in Fig. 3. These curves demonstrate the pronounced periodicity: they have maxima at the control pulse amplitudes $n\Omega_0$, where n are the integer numbers; this condition corresponds to the areas of the control pulse $2\pi n$. At these maxima, there are both the high-intensity sig-

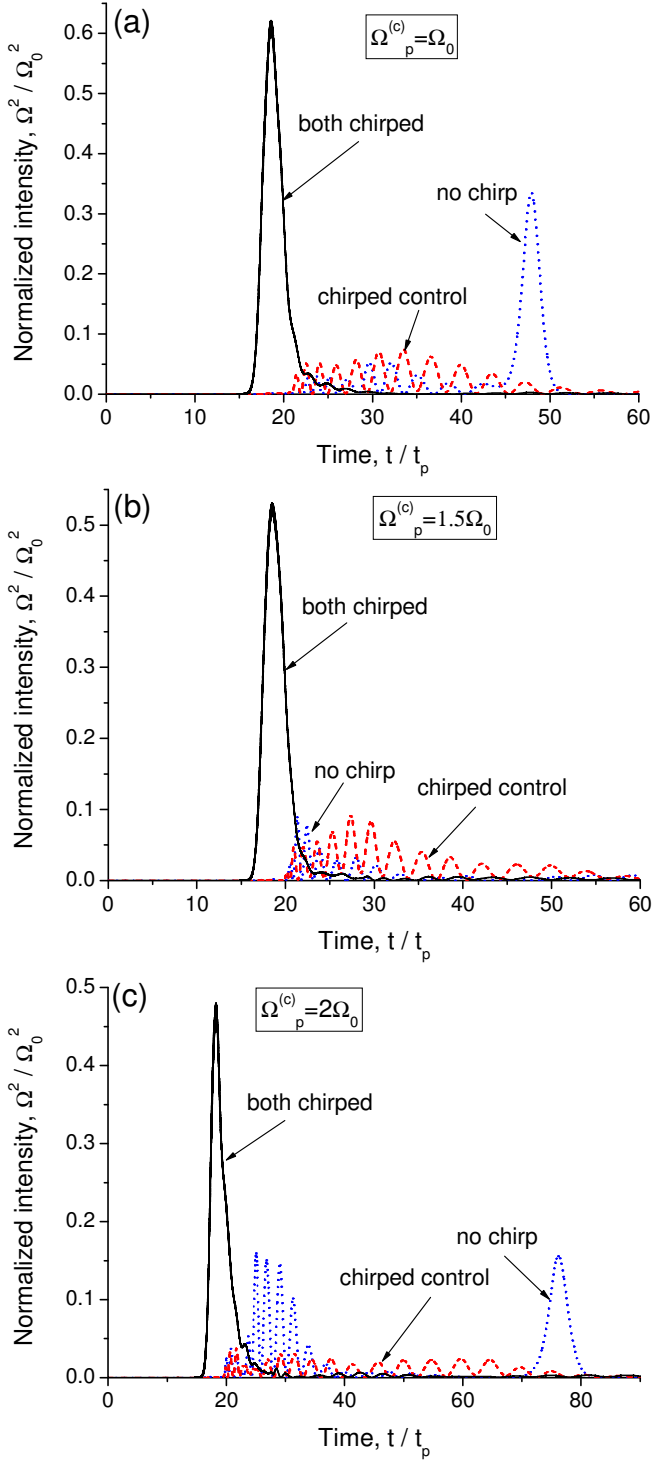


FIG. 4. (Color online) The profiles of the signal pulse after interaction with the control (counter-propagating) one. The latter's amplitude $\Omega_p^{(c)}$ is (a) Ω_0 , (b) $1.5\Omega_0$, (c) $2\Omega_0$. Calculations were performed for the same three cases and the same parameters depicted in Fig. 3.

nal soliton and low-intensity oscillations (precursor) at the output of the medium as shown in Fig. 4(a) and 4(c) (see blue dotted lines). On the contrary, at the mini-

mal peak intensity and output energy (when n is half-integer), we have only precursor at the output [4(b)], so that most part of signal energy (more than 80%) is stored inside the medium and leaves it slowly in the form of fluorescent radiation on the time scales of relaxation times. Thus, changing control pulse amplitude between integer and half-integer numbers of Ω_0 allows to switch on and off the solitonic component of the signal pulse at the medium output.

Now, let us consider the case when both signal and control pulses are chirped with $\beta = 10^{-5}$ (black squares in Fig. 3). In this case, the periodicity is much less pronounced, while the peak intensity and the output energy of the signal pulse remain relatively large at every value of the control pulse amplitude. This means that the high-intensity signal pulse only slowly changes its peak when we take different amplitudes of the control pulse as can be corroborated directly in Fig. 4 (black dashed lines). Thus, chirped pulses seem to be not suitable to effectively control one another.

The last case which we should consider is the chirp-free signal pulse interacting with the chirped control one (red circles in Fig. 3). The calculations in this case give an interesting and unexpected result: the peak intensity of the signal pulse drops to the very low values and does not manifest such sharp periodic peaks as in the case of chirp-free control pulse. At the same time, the output energy remains relatively large at all values of control amplitude. As Fig. 4 (red solid curves) shows, we have only precursor oscillations and no soliton regardless of the control pulse amplitude. This is true already for $\Omega_p^{(c)} = \Omega_0$ which means that we can use chirped control of lower intensity to get rid of chirp-free soliton and save only low-intensity oscillations. Thus, chirped control pulse appears to be very effective mean to destroy the chirp-free signal pulse.

IV. SINGLE-CYCLE PULSES

In this section, we turn to another, fundamentally different case of the pulse containing only one cycle of electromagnetic oscillations ($N = 1$). As previously, the pulse amplitude is $\Omega_p = 1.5\Omega_0$. The peak offset $t_0 = 5t_p$ is taken larger than in the previous section for the chirp to have more effect after only a single cycle. First of all, let us illustrate the importance of pulse duration for pronounced chirp influence. Figure 5 shows the dependence of the final state of inversion (the steady-state inversion established in the medium after pulse passage) on the chirp parameter β . At low chirps, we see the fully inverted state as expected in the case of 3π pulse; the final state of inversion gradually lowers as the chirp grows. One could think that the chirp effect on pulse propagation is due to the frequency shift and then make a simple estimate of the chirp parameter β_2 needed for the pulse of N_2 cycles to give the same effect as a pulse with N_1

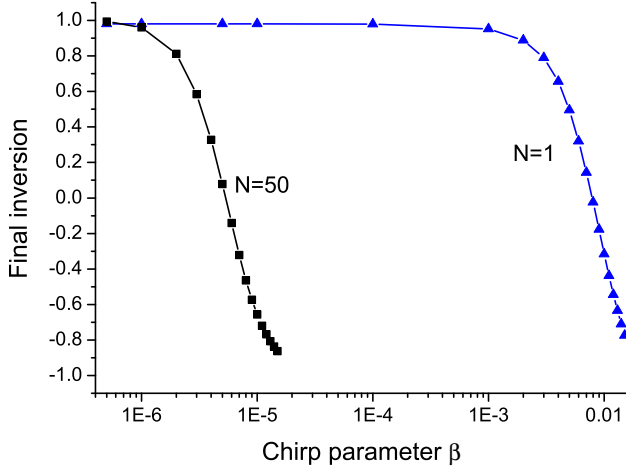


FIG. 5. (Color online) The dependence of the final state of inversion on the chirp parameter for long and single-cycle pulses. The pulse amplitude is $\Omega_p = 1.5\Omega_0$.

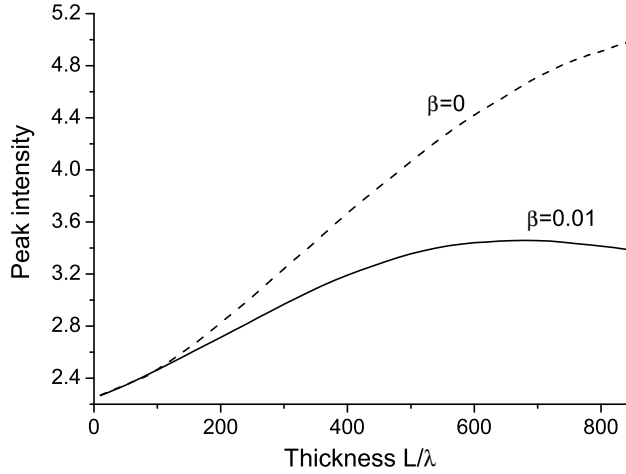


FIG. 6. The peak intensity of the chirped and non-chirped single-cycle pulses transmitted through the medium of different thicknesses.

cycles and chirp β_1 :

$$\beta_2 = \beta_1 \frac{N_1}{N_2}. \quad (4)$$

Taking $N_1 = 50$, $N_2 = 1$, and $\beta_1 = 10^{-5}$, we have $\beta_2 = 5 \cdot 10^{-4}$. Comparison with the data from Fig. 5 shows that Eq. (4) strongly underestimates the needed chirp: the final state of inversion (approximately -0.7) reached for the long pulse with parameters listed above is realized for the single-cycle pulse only at $\beta_2 \approx 10^{-2}$. Thus, in order to have strong chirp effect, we consider in this section the single-cycle pulse with the parameter as large as $\beta = 0.01$.

Next question to be discussed is the problem of pulse compression. According to Fig. 2, the long chirped pulse after transmission through the medium has larger peak intensity, i.e. it can be compressed stronger than the

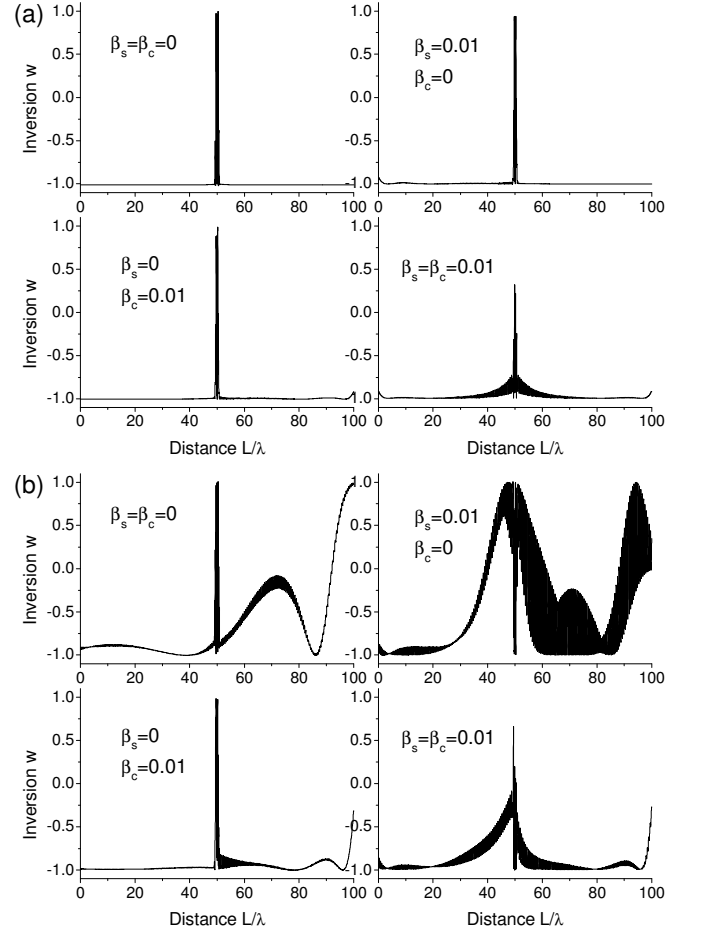


FIG. 7. Distributions of inversion along the layer of two-level medium after collision of signal and control pulses with different chirps and with amplitudes as follows: (a) $\Omega_p^{(s)} = \Omega_p^{(c)} = \Omega_0$, (b) $\Omega_p^{(s)} = \Omega_0$, $\Omega_p^{(c)} = 1.5\Omega_0$.

chirp-free one. What about single-cycle pulse? Figure 6 shows the dependence of the peak intensity on the medium thickness for such ultra-short pulses. It is seen that, contrary to the situation considered in the previous section, compression of the chirped single-cycle pulse is less effective in comparison with the chirp-free one. The reason for this is not clear. We can speculate that it may be somehow connected with the mechanism of soliton formation which can differ depending on the pulse duration.

Finally, we should analyze the situation of colliding single-cycle pulses. As previously, we call the forward and backward propagating pulses “signal” and “control” ones. As opposed to the case of long pulses, the collision of single-cycle ones does not result in any significant energy losses because of very wide spectrum of such pulses and very narrow region of their collision. Therefore, the pulses propagate almost uninfluenced after collision and there is no possibility to control their intensity and other properties. However, one can use the collisions of single-cycle pulses as a means to control the state of

the medium. Figure 7 shows an example of such control. Contrary to the collisions of long pulses where medium excitation can be reached in a certain wide spatial range, here the medium is excited in much more small volume and can be obtained using thinner layers. Consider the case of two pulses with the identical amplitudes Ω_0 [Fig. 7(a)]. As a result of collision of two chirp-free pulses ($\beta_s = \beta_c = 0$), the medium is fully inverted in a very narrow spatial interval near the center of the layer. Introducing chirp into signal or control pulse leaves the result almost unchanged. Only when both pulses are chirped ($\beta_s = \beta_c = 0.01$), the level of inversion becomes significantly lower. Obviously, changing the value of the chirp parameter, one can obtain the needed level of inversion. Temporal delay of launching of one of the pulses allows to shift the excitation region as one desires.

For the pulses with different amplitudes, the picture of collision is much more complex [Fig. 7(b)]. Generally, the distributions of inversion in this case are asymmetric with respect to the center of the layer. However, the level of this asymmetry can be controlled with the chirp. For example, the distribution in the case of chirped control pulse is almost symmetric, especially comparing with the case when both pulses are chirp-free. Thus, with the chirp, we have one more degree of freedom to control the state of the medium.

V. CONCLUSION

In summary, we have performed numerical simulations of linearly-chirped pulse propagation in the homo-

geneously broadened two-level medium. First, we have considered long (multi-cycle) pulses. In particular, we have studied transformations of such long pulses and showed that, generally, the RWA does not give a correct description of chirped pulse interaction with the medium. We have also investigated the collisions of the counter-propagating long pulses and the influence of chirp on their interaction. Our calculations have demonstrated that, if both pulses are chirped, there is only weak dependence of the signal pulse transmission on the control pulse amplitude. On the contrary, parameters of chirp-free signal can be dramatically changed with the help of the chirped control pulse. These dependencies can be used to control radiation in resonant media which seems to be perspective for all-optical logic and other applications.

Second, we have performed calculations with the single-cycle pulses and compared with the case of long pulses. We have shown that compression of the chirp-free single-cycle pulse is stronger than of the chirped one, while the opposite is true for the long pulses. As to collisions, the interaction of the single-cycle pulses can be used to control the state of the medium in the given spatial region but not the state of the pulses themselves. The chirp gives the additional possibility to change the spatial distribution of medium inversion which can be used for the all-optical precise control of medium excitation level.

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